2009-2010 Puzzle of the Month Contest

Solutions for Contest #3

Parents and Grandparents Puzzle Solutions:

1. THE LAST NEWS FROM THE MARS

The first meeting of the Martians and the People had revealed that the Martian's legs were the same as the People's. But the number of hands and the number of fingers on a hand were not the same as the People have. After full calculation, the total number of the People's fingers and toes were 1 more than the total number of the Martian's fingers and toes, although there were 6 more Martians than People. How many participants were there on the meeting? (Thumbs are also counted as fingers, all Martians have the same number of hands and fingers, and all people have the same numbers of hands and fingers)

50 points

Answer: ***236 participants = 115 of the People +121 of the Martians***

2. THE "USELESS" FACTORIAL

***Remove* one of the factorials** from the product $1! \cdot 2! \cdot 3! \cdot \ldots \cdot 99! \cdot 100!$ so that the product of the remaining factors forms the perfect square (N! is a product of all natural numbers from 1 to N: $N! \equiv 1 \cdot 2 \cdot 3 \cdot \ldots \cdot N$). **20 points**

Answer: $50!$ *has to be deleted.*

Proof: $1! \cdot 2! \cdot 3! \cdot \ldots \cdot 99! \cdot 100! = 2(1!)^2 \cdot 4 \cdot (3!)^2 \cdot \ldots \cdot 98 \cdot (97!)^2 \cdot 100 \cdot (99!)^2 =$
$= 2^{50} \cdot (50!) \cdot (1!)^2 \cdot (3!)^2 \cdot (5!)^2 \cdot \ldots \cdot (99!)^2 = (50!) \cdot (2^{25} \cdot 1! \cdot 3! \cdot 5! \cdot \ldots 99!)^2$.

Hence (50!) has to be deleted for getting a perfect square.

3. THE MYSTERIOUS TRIANGLE

Some natural numbers are written in the vertices of a triangle. The product of two numbers in the adjacent vertices is written on the side that connects them. The product of all vertex numbers is written inside the triangle. The total sum of all 7 numbers is equal to 1000. What are numbers written in the vertices of the triangle? **30 points**

Answer: x=6, y=12, z=14.

Solution: Let x, y, z be numbers which are written in the vertices of the triangle. Then x·y, y·z, z·x are written on the sides. The number x·y·z is written inside the triangle. As per the problem statement, the total sum of them is equal to 1000. We can write the equation : $x + y + z + x \cdot y + y \cdot z + z \cdot x + x \cdot y \cdot z = 1000$

or $1 + x + y + z + x \cdot y + y \cdot z + z \cdot x + x \cdot y \cdot z = 1001$.

To factor the left side of equation: $(1 + x)(1 + y)(1 + z) = 1001 = 7 \cdot 11 \cdot 13$.

Hence the answer is $x = 6, y = 10\not{2}, z = 124$ *or any permutation of these numbers.*