2009-2010 Puzzle Contest #5
Solutions

Grades 5 and Up Puzzle Solutions:

1. AN IMPROVIDENT SNAIL

A SNAIL CREEPS ON THE GROUND starting from a point A, turning *90°* right or left every *15* minutes. Can it return to the starting point *A* exactly in *17.5* hours? Explain your answer (The speed of the snail is considered constant, every snail's 15-min move is a line segment, assume the first *15*-min move is in the horizontal direction(along X-axis)). **20 points**

Answer: For any way a snail would not be able to return to the point A in 17.5 hours accurately

Solution: *Denote:*

1) *because of snail's properties there is exactly one 15-min horizontal move for each vertical one.*
2) *the number of 15-min moves for both horizontal directions(from point A and to point A) has to be the same*
3) *the number of 15-min moves for both vertical directions(from point A and to point A) has to be the same.*

Let be n –the number of 15-min moves along horizontal direction from point A. Because of 1), 2), and 3) total number of 15-min moves from point A and to point A is $4n$. *Total time of snail's trip is* $4n \cdot 15\ min = 60n\ min = n\ hours$. *It means that the total time can't be with fractional hours.*

2. ABOUT ONE SPECIAL NUMBER

IN A DECIMAL NOTATION of a number there are 309 ones and all the other digits are zeros. Could this number be a a perfect square of another number? **50 points**

Answer: No, it can not be

Solution: Assume this number N could be written as a perfect square m, i.e. $N = m^2$. However N is divisible by 3 because the total sum of all digits of this number is 309 that is divisible by 3. Then m is divisible by 3.

Hence

$N = m^2$ *has to be divisible by* 9. *But* N *is not divisible by* 9 *because* 309 *is not divisible by* 9.

This contradicton proves the answer

3. WIN OUT !

SOLVE THE CRYPTOGRAM: $7 \cdot \overline{WINOUT} = 6 \cdot \overline{OUTWIN}$, where W, I, N, O, U, and T are non-zero digits of 6-digit numbers **30 points**

Solution: Let $\overline{WIN}$ =x and $\overline{OUT}$=y. Then $7(1000x + y) = 6(1000y + x), 6994x = 5993y, or\ 538x = 461y$.

Because these numbers 538 *and* 461 *are mutually prime,*

the last equlity $\boldsymbol{x = \overline{WIN} = 461\ and\ y = \overline{OUT} = 538}$