



2009-2010 Puzzle Contest #3 Solutions



Grades 5 and Up Puzzle Solutions:

1. THE REBUS FOR KIDS

Does the rebus $KI \times DS = MAMA$ have a solution? Explain your answer.

(Each letter is a decimal digit. Different letters are different digits) **30 points**

Answer: No, it doesn't. $MAMA = 101 \times MA = KI \times DS$. One of two last factors has to be three-digit number. This is because 101 is a prime number, so either KI or DS must be divisible by it. But both of them are two-digit numbers.

2. THE HIDDEN NATURAL NUMBER

Connor had calculated the sum $1+2+3+\dots+N$. After taking away three right most digits of it he got the number N again. Find N . **20 points**

Answer: $N=1999$. Solution:

$1 + 2 + 3 + \dots + N = \frac{N(N+1)}{2}$. Taking away three youngest digits means the following:

1. $\frac{N(N+1)}{2}$ has to be divided by 1000

2. the remainder is less than 1000 and it has to be taken away.

Consider $N \geq 1999$

Let r be a remainder. Then $\frac{N(N+1)}{2} = 1000 \cdot N + r$ or $N(N+1) = 2 \cdot 1000 \cdot N + 2 \cdot r$.

Or $N+1 = 2000 + \frac{2r}{N}$.

Since we have the whole number in the left side of the equation m has to be equal zero. Hence $N+1 = 2000$ or $N = 1999$

3. THE HARD-WORKING CAMEL

In city A in a desert, there is a man who owns a camel. The man needs to take 1000 watermelons to a fair in city B where he wants to sell the watermelons. The distance between A and B is 7 km. However, the camel can carry a maximum of 100 watermelons at a time. In addition to that, the camel eats one watermelon per each kilometer it travels.

The man wants to bring to the fair as many watermelons as possible. How can he do it and what is the maximum number of watermelons he can bring to the fair? **50 points**

Answer: 869 watermelons

If the distance were 1km, the obvious maximum would be 981 (as the camel needs to travel 19 times). If the distance were 2 km, the result would be same as if the man had 981 watermelons and to go only 1km, so the result would be 962. (Note: The result for distance of 6 km is 886 but from this moment the camel needs to travel only 17 times to get all the watermelons 1km further, so the result for the distance of 7km is 869)